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ABSTRACT

This paper compares the mathematical understandings and pedagogical content knowledge of beginning teachers entering teaching through an alternate route program with those entering from three standard teacher education programs. The analysis challenges two common assumptions about becoming a secondary school mathematics teacher: (1) that people who major in mathematics without emphasis on education are more capable and know more than their mathematics education peers; and (2) that professional knowledge and pedagogical content knowledge are best acquired through practical experience as a teacher, university-based teacher education being unable to make practical or significant contributions to what teachers need to know or be able to do. Yet, despite structural and philosophical differences between university and alternate route programs, the novice teachers across the two groups were much the same. Neither the teacher education students nor the teacher trainees in the sample were well prepared to unpack meanings of mathematical ideas on the basis of their studies. Neither program had any consistently strong impact on novice teachers' ideas about the teacher's role or about desirable practices in teaching mathematics. Many in both groups were unable to represent basic content in meaningful ways at the end of their programs. The supposed advantage of teaching experience in the case of the alternate route teachers did not emerge as a significant factor in this study. (Author/JD)

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Research Report 90-7

Knowing the Subject and Learning to Teach It: Examining Assumptions About Becoming a Mathematics Teacher

Deborah Loewenberg Ball and Suzanne M. Wilson

NCRTE

National
Center for Research
on Teacher Education

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Research Report 90-7

KNOWING THE SUBJECT AND LEARNING TO TEACH IT: EXAMINING ASSUMPTIONS ABOUT BECOMING A MATHEMATICS TEACHER

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Abstract

This paper compares the mathematical understandings and pedagogical content knowledge of beginning teachers entering teaching through the Los Angeles Unified School District alternate route program with those entering from standard teacher education programs at three universities and colleges. The analysis challenges two common assumptions about becoming a secondary school mathematics teacher, assumptions that underlie the development of alternate routes to certification. The first assumption is that people who major in mathematics without an emphasis on education are both more capable and know more than their mathematics education peers. The second is that professional knowledge and pedagogical content knowledge are best acquired through practical experience as a full-fledged teacher, that university-based teacher education can make few practical or significant contributions to what teachers need to know or be able to do. Yet, despite apparently dramatic structural and philosophical differences between university-based and alternate route programs, much was the same about the novice mathematics teachers across the two groups in this analysis. Neither the teacher education students nor the teacher trainees in our sample were well-prepared to unpack meanings of mathematical ideas on the basis of their mathematical studies. Neither did either group change significantly during their programs. The overall proportions of novice teachers with conceptual understandings of elementary mathematics topics were discouraging. Furthermore, neither the teacher education programs nor the alternate route had any consistently strong impact on novice teachers' ideas about the teacher's role or about desirable practices in teaching mathematics. Many novice teachers in both groups were still unable to represent basic content in meaningful ways at the end of their programs. The supposed advantage of teaching experience in the case of the alternate route teachers did not emerge as a significant factor in our analyses.

KNOWING THE SUBJECT AND LEARNING TO TEACH IT: EXAMINING ASSUMPTIONS ABOUT BECOMING A MATHEMATICS TEACHER[1]

Deborah Loewenberg Ball and Suzanne M. Wilson[2]

Introduction

Consider the domain of traditional university-based teacher education: Prospective teachers typically take courses in learning and human development, general methods of teaching, subject-specific teaching methods, and social foundations of education. Also required is a term of supervised student teaching, sometimes complemented by other clinical experiences. In addition, prospective secondary teachers complete a disciplinary major, while elementary teacher candidates' studies are usually broader and unspecialized.

Among policymakers, traditional teacher education is under fire for a number of reasons, most notably for its failure to recruit academically strong candidates and its perceived lack of effects. Critics claim that university-based teacher education tends to attract only mediocre students whose knowledge of the subject matter is often weak. And many believe that professional education makes little difference, for learning to teach, they argue, occurs primarily through experience. An additional specific problem is the overwhelming two-way shortage of teachers--in urban areas and in mathematics and science. Teacher education is in need of reforms that will attract a population of qualified mathematics and science teachers who can work effectively with at-risk students in inner city schools.

Concerns for these problems have spawned creation of a number of "alternate route" teacher certification programs. These state and district-level programs enroll college graduates and provide them with brief intensive training. Equipped with information about topics as various as lesson planning, techniques of classroom management, and working with non-English-speaking students, the new teachers dive into the job. Throughout their first year, they receive additional guidance--evening seminars on particular topics or techniques and, in some cases, in-class support in the form of a mentor or supervisor.

We acknowledge that these cursory characterizations of both traditional teacher education programs and alternate routes do not recognize the vast differences that exist across programs for teacher preparation. Not all alternate routes are alike, nor are all traditional teacher education programs. However, in this paper we focus on the underlying

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[1]This paper was presented in April 1990 at the annual meeting of the American Educational Research Association in Boston.

[2]Deborah Loewenberg Ball and Suzanne M. Wilson, assistant professors of teacher education at Michigan State University, are senior researchers with the National Center for Research on Teacher Education. The authors gratefully acknowledge other members of the research staff of the NCRTE for their contributions to instrument design, data collection and analysis; in particular: Marianne Amarel, Sandra Callis, Ada Beth Cutler, Robert Floden, Mary Kennedy, G. Williamson McDiarmid, Harold Morgan, Barbara Neufeld, Michelle Parker, Jeremy Price, Pamela Schram, Trish Stoddart, and Lauren Young.

assumptions of these programs, many of which are shared across programs of teacher preparation. We focus our discussion on two such assumptions.

First, it is assumed that college graduates who majored in liberal arts have, in general, more subject matter knowledge than do their teacher education counterparts. With a college education uncluttered with professional course work, they are thought to have had a more concentrated training. Furthermore, many believe that these individuals are academically more capable--after all, only those who cannot succeed in traditional academic disciplines would choose to teach. Valuing a stronger background in the root disciplines of school subjects, policymakers assume that teachers need to know the subject matters they teach and that the knowledge of subject matter necessary for teaching can be acquired through traditional undergraduate course work in the relevant disciplinary department.

Second, it is assumed that subject matter knowledge is the only professional knowledge one needs to acquire in formal university or college settings. Other types of knowledge and skill necessary to teaching, e.g., pedagogical knowledge or pedagogical content knowledge, can and should be acquired through practical experience as a full-fledged teacher. Immersion in the reality of the job--supported by mentors, supervisors, and ongoing seminars--is assumed to be an effective way to learn to teach. Traditional teacher education is thought to make few practical or significant contributions to what teachers need to know or be able to do.

These assumptions are embedded in a complex web of other assumptions and beliefs about teaching and learning to teach. Rather than explore this web here, we focus on these two beliefs and examine their validity in light of data we have collected on two groups of novice secondary school mathematics teachers. The first group consists of 22 undergraduate students preparing to teach and majoring in mathematics at Dartmouth College, Illinois State University, and Michigan State University; the second group, 21 postbaccalaureate mathematics majors entering teaching through the Los Angeles Unified School District alternate route program.

Method

This paper draws on data from the Teacher Education and Learning to Teach study (TELT) at the National Center for Research on Teacher Education (NCRTE) at Michigan State University. The study examines what teachers are taught and what they learn in 11 diverse preservice, induction, inservice, and alternative-route programs around the country, combining case studies of programs with longitudinal studies of participants' learning (Ball and McDiarmid, 1988; NCRTE, 1988). The study design is longitudinal. At repeated intervals, we administer a questionnaire to all the college students in the sample; we also interview and observe a smaller "intensive" sample of students whom we follow more closely

by interviewing and observing them throughout their preservice program and into their first year of teaching.

Both the questionnaire and the interview were designed to explore participants' ideas, feelings, and understandings about mathematics and writing, about the teaching and learning of mathematics and writing, and about students as learners of these subjects. Many of the questions are grounded in scenarios of classroom teaching and woven with particular subject matter topics. Among the mathematical topics are rectangles and squares, perimeter and area, place value, subtraction with regrouping, multiplication, division, fractions, zero and infinity, proportion, variables and solving equations, theory and proof, slope and graphing.

For the analysis reported in this article, frequencies were calculated for relevant questionnaire responses. Careful substantive analyses of the interview questions led to the creation of a set of response categories for each one, as well as a set of categories for the types of change we observed in respondents' answers over the course of their programs. These categories were modified in the course of data analysis to better accommodate participants' responses. The questionnaire and interview responses were then examined in light of the two assumptions we discuss here: (1) what subject matter knowledge counts in teaching and (2) how people acquire knowledge of pedagogy. We explore each in turn.

Assumption #1: Liberal arts majors understand their subjects well--better than education majors

Policymakers and the public alike doubt the adequacy of many beginning teachers' subject matter knowledge. They do so with some reason, for evidence suggests that teacher education students are often weaker academically than their liberal arts peers (e.g., Carnegie Task Force, 1986; Lanier with Little, 1986). Consequently, alternate route programs are appealing, because they are thought to attract well-educated graduates who understand subject matter better. Both because of what they study and their generally higher intellectual caliber, it is assumed that the liberal arts majors who enter teaching through such routes have better understandings of their subjects than do education majors.

Counter to this assumption about the greater subject matter knowledge of individuals in the alternate route programs, the results from our interviews and questionnaires indicate little difference in the mathematical understandings between novices in the alternate route program and teacher education candidates, either when they enter their program or when they finish. In Tables 1 and 2 we present the results from our analyses of several questions in the interviews that were designed to tap the subject matter knowledge of the respondents. The questions were as follows:[3]

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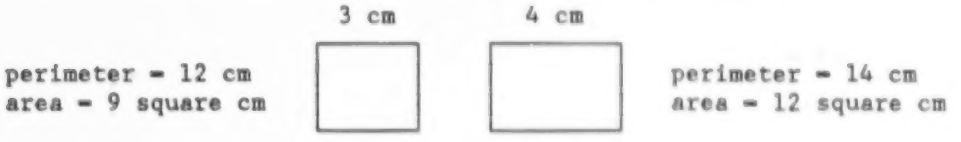
[3]All three of these questions were asked during Wave 1 (program entry) and Wave 2 (completion of program), although some details were changed slightly between versions. For each of the mathematics questions discussed in this paper, we provide a brief explanation of the underlying mathematical ideas. For a more complete explanation of the rationale and conceptual frame of the NCRTE interview questions, see Ball & McDiarmid, 1989.

1. Imagine that one of your students comes to class very excited. She tells you that she has figured out a theory that you never told the class. She explains that she has discovered that as the perimeter of a closed figure increases, the area also increases. She shows you this picture to prove what she is doing:[4]



2. Division by fractions is often confusing. People seem to have different approaches to solving problems involving division with fractions. Something that many teachers do is relate mathematics to other things. Sometimes they try to come up with real-world situations or story problems to show the application of some particular piece of content. What would you say would be a good situation or story or model for

$$1\ 3/4 \div 1/2?$$[5]

3. Suppose you have a student who asks you what 7 divided by 0 is. How would you respond?[6]

These three questions require respondents to assess the correctness of mathematical ideas, as well as to explain and elaborate meanings underlying some conventional procedures and ideas that they have learned (e.g., that "you can't divide by zero"). Our questions sought

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[4]There are two dimensions to the question. The first concerns the specific concepts of perimeter and area and their relationship, the substance of the pupil's claim. The second concerns mathematical knowledge and its justification--what constitutes "proof" of a generalization in mathematics. The pupil claims to have discovered a "theory" and offers a picture of a single case as a "proof." An example, however, does not establish the truth of a generalization in mathematics; even many examples would only make the claim more plausible, but not prove it. Hence, the pupil's claim is a conjecture, not a theory. And, in fact, it is false, for the perimeter and area of a closed figure increases in direct relationship only if the figures are similar (e.g., two circles, two squares). In this case, the 3 x 3 rectangle has a perimeter of 12 cm and an area of 9 square cm. If the rectangle's dimensions were changed to 1 x 9, the perimeter would be greater (20 cm) while the area would remain the same (9 square cm).

[5]Fraction division is no different in meaning from whole number division. One interpretation of 1 3/4 ÷ 1/2 is "how many halves are there in 1 3/4?" One appropriate story might be, "How many half-cup scoops will it take to measure out 1 3/4 cups of flour?" Most people, whose only experience with fraction division is the algorithm "invert and multiply," produce stories or situations that model division *in* one-half instead of *by* one-half: "I have 1 3/4 pizzas and I want to share the pizza equally between two people. How much pizza would each person get?"

[6]Division by zero is undefined because it does not fit the definition of division: b is divisible by a if there is some number c such that $a \cdot c = b$; for example, 12 is divisible by 3 because $3 \cdot 4 = 12$. With division by zero, there is no number that can be multiplied by zero to equal the dividend; for example: 7 is not divisible by 0 because there is no number times 0 that can equal 7. Another way to look at it is to notice that, as the divisor gets smaller (approaches zero), the quotient gets larger, so it explodes and is undefined at 0 (e.g., 7 ÷ 1/2 = 14, 7 ÷ 1/4 = 28, 7 ÷ 1/100 = 700, and so on).

Table 1

Interview Data: Percentages of Alternate Route Trainees and Teacher Education Candidates Who Evidence Mathematical Understandings at Program Entry

	Sees falsity of student's claim about the relationship between perimeter and area	Sees insufficiency of proof by example	Can explain why division by zero is undefined	Can represent division of fractions to show what it means to divide by a fraction
Alternate route $N = 7$	57	13	43	14
Teacher education $N = 12$	40	23	42	42

Table 2

Interview Data: Percentages of Alternate Route Trainees and Teacher Education Candidates Who Evidence Mathematical Understandings at Program Exit[a]

	Sees falsity of student's claim about the relationship between perimeter and area	Sees insufficiency of proof by example	Can explain why division by zero is undefined	Can represent division of fractions to show what it means to divide by a fraction
Alternate route $N = 12$	43	38	86	42
Teacher education $N = 7$	68	30	59	49

[a]Where percentages drop from Table 1 (program entry) to Table 2 (program completion), this is due to a few novices who "unlearned" what they knew when they came. This is probably best explained by assuming that, at the outset, their understandings were fragile and unreliable and that they remained so.

to probe below the surface of conventional displays of mathematics knowledge--that is, merely providing right answers. We also probed for how they understood the content. Did they understand why an answer made sense or had they just memorized it?

Our results suggest that neither group could explain the underlying mathematical meaning of the procedures and ideas presented in these three problems. While there were times when the alternate route teacher trainees[7] seemed to have an edge (e.g., explaining why division by zero is undefined), there were an equal number of times when the teacher education majors seemed to have a clearer sense of the mathematical meaning of a problem (e.g., representing the meaning of division by fractions).

While there appear to be few differences between the groups, it is distressing how little either group knows about the mathematics of these common school problems. With the exception of explaining $7 \div 0$, less than half of our novice mathematics teachers--in teacher education programs or alternate routes--could critically examine the problems presented to them and convincingly answer questions concerning the mathematics represented in those problems. Similar trends were seen in our analyses of the questionnaire data although the percentages are higher. This is not surprising given the fact that questionnaire respondents had only to select from an array, the correct response, rather than generate one on their own (see Table 3).

These results were distressing to us not only for what they revealed about the mathematics majors' knowledge of elementary topics, but also because the novice teachers were often aware that all they had learned were rules and procedures, that no one had helped them develop meaningful understandings of the tools they had memorized and learned to use in algorithmic ways. One teacher trainee reflected, "How can I teach if I don't know why myself?" And one of the teacher education mathematics majors remarked ruefully,

> Here I am, it's weird, here I am supposedly this math wizard and I've got a lot of knowledge and I've probably made more connections than a lot of people, but there are a lot of connections that I haven't made. I haven't seen these things before and I don't know where I was supposed to learn them--in high school? Or middle school?

Our data suggest that it may be problematic to assume that mathematics majors understand underlying meanings for mathematical ideas. This is easily understandable when one considers the kinds of mathematics classrooms where they have acquired their

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[7]For ease of discussion here, we will refer to the novices in the alternate route program as teacher trainees and those in the traditional programs as teacher education majors or students. By making this distinction, we do not mean to suggest that one group is being trained and the other educated, for both groups are learning a great deal from their experiences. We use the language simply to help the reader distinguish between the groups to which we are referring.

knowledge and skills. At all levels of schooling, the patterns of mathematical pedagogy revealed by research are characterized by a consistent emphasis on memorization and rules, fast-paced content coverage, and getting right answers (e.g., Goodlad, 1984; Stodolsky, 1988). In other words, mathematics majors have had predictably few opportunities to unpack mathematical ideas or to make connections. And whether they have undergraduate degrees in mathematics or in mathematics education, students of mathematics have not been exposed to the alternative models for teaching mathematics. Thus, neither group is prepared to teach mathematics in ways that emphasize meaning.

Assumption #2: If teacher trainees have the necessary subject matter knowledge, experience in schools can teach them everything else they need to know

A second major assumption made by supporters of alternate route programs is that what well-educated people need to know about teaching can be learned through experience, supplemented with a some important techniques and information that can be effectively provided in summer workshops and evening seminars. Well-grounded in the subject matter, it is assumed that these individuals are better equipped to learn to transform the important ideas of their field so that students learn. Add a little experience to a strong knowledge of the subject matter, so the assumption goes, and teachers will eventually acquire the necessary generic and subject-specific pedagogical knowledge.

To analyze the validity of this assumption, we examined two aspects of the novice teachers' developing ideas about teaching: their notions about the teacher's role in helping students learn about mathematics, and their pedagogical perspective on the content. As for role, we saw little difference in their views as a group: At the beginning of their programs, we presented questionnaire respondents with four alternative teachers and asked them who would be the one most likely to help students learn mathematics. When they entered their programs, both groups thought that a teacher who shows and tells students exactly how to do the work is most likely to help students learn mathematics. A little over half of the teacher trainees in the alternate route entered the program expressing a preference for this directive teaching style; fewer of the teacher education students were convinced of this, but it was still favored more than any other teaching style. By the end of the program, more novices in both groups had shifted to describing good teaching in terms of "leading" and "guiding" students rather than telling (see Table 4). But in neither group did more than one or two people favor a more facilitative, constructivist-oriented style.

We saw similar trends in responses to interview questions in which we posed pedagogical problems (e.g., a student suggests a nonstandard algorithm, asks a question, presents an error on a paper) and asked respondents how they would deal with the situation. In every case, teacher candidates and teacher trainees alike said they would respond directly to the student, telling the student if the idea was correct, showing him or her what to do,

Table 3

Questionnaire Data: Percentages of Alternate Route Trainees and Teacher Education Candidates Who Evidence Mathematical Understandings at Program Exit[a]

	Can select an appropriate representation for a division of fractions problems	Can see mathematical sense of a nonstandard procedure for borrowing[b]	Believes that the principle "a negative times a negative equals a positive" can be explained meaningfully[c]	Believes that the rule that "any number to the power of 0 equals 1" can be explained meaningfully.
Alternate route $N = 21$	67	70	75	85
Teacher education $N = 22$	57	81	64	54

[a]Important to note is that the questionnaire provides qualitatively different information about participants' understandings. For division of fractions, they were asked to select appropriate representations from among an array of choices. This is an easier task than generating an appropriate representation on one's own. And, for the other three items reported in this table, participants were asked whether or not something made sense or could be explained meaningfully, not to actually produce a mathematical explanation.

[b]A student invents a "simpler method" to use in place of the traditional borrowing. She shows you:

$$\begin{array}{r} 36 \\ -\ 19 \\ \hline -\ 3 \\ \hline 20 \\ \hline 17 \end{array}$$

She explains that 6 - 9 equals -3 and 30 - 10 = 20 = 17. This makes mathematical sense because numbers can be decomposed in this way and then recombined to arrive at an answer: 36 = 30 + 6 and 20 = 20 + 0, and it makes sense to work with the ones and then work with the tens and then put the number "back together."

[c]Note that this item and the next one asked participants to decide whether a particular algorithm or rule *could* be explained meaningfully or whether it had to just be memorized--but we did not ask people to provide such explanations on the questionnaire. The data therefore tell us only whether the person *believes* that the rule in question *could* be explained, not whether they themselves could actually provide such an explanation. In other words, we would predict that these results would probably overpredict the percentages of people who could explain the rules given. In all cases of items like this, the particular rules given were things that *could* be explained conceptually.

Table 4

Percentages of Alternate Route Trainees and Teacher Education Candidates at Program Exit Who Select Particular Models of Successful Teachers

	CASS	JAY	SAM	RANDY
Alternate route *N* = 21	10	35	30	25
Teacher education *N* = 20	5	24	47	24

CASS: facilitator of individual student construction
JAY: leading and guiding with questions
SAM: orchestrating group discussions focused on meaning
RANDY: direct instruction

Table 5

Percentages of Alternate Route Trainees and Teacher Education Candidates at Program Exit Who Evidence Tendencies to Explain or Tell Students Directly

	Traditional teacher role unchanged in responding to novel student conjectures	Traditional teacher role unchanged in responding to students' questions or puzzlements
Alternate route *N* = 20	88	100
Teacher education *N* = 22	92	100

answering questions directly. And there was virtually no change in this over the course of their programs (see Table 5). In other words, the traditional images of good mathematics teaching which our respondents entered their programs with--images deeply rooted in their experiences as students--did not seem to change significantly as a result of their professional training of either kind.

One interesting example was the novice teachers' views about teaching rules of thumb in mathematics. We asked them, on the questionnaire, whether teaching "rules of thumb" (e.g., "any number divided by itself equals 1," "you can't take a smaller number away from a larger one") was, in general, a good idea.[8] Alternative answers provided on the questionnaire were as follows:

1. I think these rules help students to remember basic principles that they need to have at their fingertips.

2. I think these are a bad idea--they give the impression that mathematics is a set of procedures to be memorized.

Approximately two-thirds of both groups--the trainees and the teacher education majors--favored teaching rules of thumb when they entered their programs and there was little change by the time they left their programs. Table 6 shows the distribution of views on this issue. Thus, in both groups, at least one view about traditional good practice was left relatively untouched by either standard or alternative teacher education.

Another critical dimension of learning to teach is developing pedagogical knowledge of the content. One aspect of this means acquiring knowledge of how to represent mathematical ideas to students in ways that will bridge between their prior and current knowledge and the mathematics they are to learn. So, for example, when teaching division by fractions, good teachers know that students who can chant "invert and multiply" may not understand the differences between dividing by 2 and dividing by one half. Or when a student invents a novel way of doing a problem or asks an unusual question, good teachers are able to discriminate between valid and invalid notions, between important and trivial points, so that they can make good decisions about how to respond to students. Teachers who themselves are tethered to a procedural knowledge of mathematics are ill-equipped to deal with student questions and frustrations about why inverting and multiplying works,

[8]Besides the issue of portraying mathematics as a set of rules to be memorized, another issue entailed in "rules of thumb" is that their truth is usually limited to a particular mathematical context. For example, subtracting a larger number from a smaller one *is* possible in the integers, just not in the counting numbers. Some rules of thumb even gloss over exceptions--e.g., any number divided by itself is 1--*except for 0*, for 0/0 is indeterminate. On the other side of the issue is the helpfulness of easy-to-remember catchy principles.

about why zero divided by zero is not equal to 1, or about why division by zero is "forbidden."

Consider the question concerning representing division of fractions, and the difficulties both groups had in answering questions about real world examples of such problems. Less than 50 percent of either group could successfully generate representations of this idea, yet such examples are exactly the kinds of tools teachers might draw upon in helping students understand the concept of division by fractions. The teacher education students who could generate representations for the problem would offer problems as Molly did: "You could say that we have 1 3/4 pizzas and we're having a party. Each person is going to get half a pizza. How many people can we invite?"

More than half of our respondents could not generate such examples. Some were completely baffled and considered the problem "obscure" like Carl, a teacher trainee in Los Angeles:

> I don't know why it works. I mean, even to this day, I have one hell of a time working with fractions. I just convert everything to decimals and plug it into my calculator. . . . Why would you even *want* to divide something by a half?!

Gillian, a teacher education student, offers another example of a teacher who has trouble making the problem concrete:

> I don't know. I think you would have to talk in big numbers so that they would understand. Like perhaps talk about the impact of a bomb. If the world was so big and the bomb was let off, it would divide the world. Then how many pieces would be left or something like that. It's hard to find an application for these specific numbers.

Or consider Geoffrey's comments. He is sure there is no real world representation of division by fractions:

> Generally you don't divide by fractions. I mean, you do a lot if you're talking about real world people who do math as an occupation. But I'm thinking about apples and oranges and things like that--pieces of pie. Who divides by a half with that?

Other teacher education students, using their knowledge of the invert-and-multiply algorithm, tried to generate representations that matched the shortcut. These responses were extremely muddled and one wonders how students would react to such explanations. Consider Michelle's response:

Table 6

Percentages of Alternate Route Trainees and Teacher Education Candidates at Program Exit Who Think It Is a Good Idea to Teach Students Rules of Thumb in Mathematics

	Teaching rules of thumb is a good idea	Teaching rules of thumb is a bad idea
Alternate route *N* = 20	80	20
Teacher education *N* = 22	72	28

> There's this gremlin and he is 1 3/4 feet tall. And he has this friend and his friend is half as tall as he is. If they think about it, gremlin B is twice as tall as gremlin A. And if you think of twice as tall, that's the same as dividing by a half.

Still other novice teachers were quite articulate as they offered representations, but failed to recognize that their model was for the problem 1 3/4 ÷ 2 instead of 1 3/4 ÷ 1/2. Carson, a teacher trainee, provides a good example of a clearly put, but completely inaccurate representation of the problem: "We had a party and we have one and three quarters pies left over and I have to divide it between two people. How do I do that? How much does each person get?"

Similar patterns appear in the responses of novice teachers to other questions concerning the representation of mathematics to students in explanations. When pushed to explain why division by zero is undefined, for example, many respondents found themselves struggling to invent an explanation for something that they had always simply accepted as "fact." Camille, a teacher trainee in Los Angeles, explains:

> You can't divide by zero. I'm not sure that anyone really knows why. I don't really know why, I just know you can't. . . . I would tell [students] that it would be undefined in math. There are a few things in math that are not given any reason for why you do it, and one of them is that you can't divide by zero.

Some respondents tried to develop representations that would make sense to students but failed to make the representation mathematical. For instance, Gillian's explanation relied on the use of eggs:

> It's like something on top of an egg. If you put a box on top of an egg and ask, "If the box is any heavier, could this work?" Try to get it into their minds that you cannot have something on top of zero. Zero would be represented by an egg and it would go splat.

Still other novice teachers, assuming there was an explanation, nevertheless got confused as they tried to explain it in the interview. Gerald, for instance, remarked:

> When you divide nothing into something, you can't. You're getting nowhere. I'd try to tell [the student] that it's like when you try to put nothing into something, you're not putting anything in there . . . you just keep beating a dead horse. You're just putting nothing into nothing and you're getting nowhere.

Frustrated by his inability to be articulate about the reasons, Gerald concluded his remarks by saying, "It's a hard question. I don't know where I'd go after [I tried] that." Again, one wonders what sense students would make of such explanations or what these novice teachers would do in the face of the frustration they would feel when unable to offer answers to students questions about why something works mathematically. Images of Camille's "explanation"--"there are just some things in mathematics that have no reason for why you do it"--haunt one in studying these novice teachers' attempts to represent mathematical ideas and their potential for explaining these ideas to students.

With no significant differences between the two groups of novice teachers--upon entry into or exit from their respective programs--their ability to offer mathematical explanations to students is discouraging. The supporters of alternate routes may be right in their assumption that prospective teachers do not learn the appropriate things about teaching in their traditional teacher education programs, but our analyses suggest that neither group is prepared to teach mathematics for understanding nor to teach mathematics in a way that differs from the traditional pedagogy of telling and drilling algorithms into students. One novice teacher expressed this view succinctly when he described how he would help a student, "I'd just keep trying to pound that one into him."

Where is the Impact? Little "Alternative" in the Alternative: The Challenges of Becoming a Mathematics Teacher

One attraction of alternate route programs is the widespread belief that traditional teacher education does not recruit promising candidates and makes little difference with those whom it does recruit. Teacher education courses are thought to have little impact on prospective teachers' knowledge or skills. And the reputation of teacher education as an intellectual wasteland does little to interest bright people who would like to teach. Alternate routes, therefore, represent an effort to provide essential knowledge under efficient and effective conditions, thereby also removing recruitment obstacles.

Our data raise questions, however, about the assumed differences in becoming a mathematics teacher through university-based teacher education and an alternative route to teacher certification. We see problems inherent in assuming what people who have majored in mathematics know. Neither group was well-prepared to unpack meanings of mathematical ideas; neither did either group make huge changes during their programs. In fact, over half of those who entered the programs unable to generate an appropriate representation for division of fractions or unconcerned with issues of mathematical proof left the programs still lacking these understandings. The overall proportions of novice teachers with conceptual understandings of elementary mathematics topics were discouraging.

Neither these teacher education programs nor the alternate route had any consistently strong impact on novice teachers' ideas about the teacher's role or about desirable practices

in teaching mathematics. And, considering the importance of developing pedagogical content knowledge for teaching, many novice teachers in both groups were still unable to represent basic content in meaningful ways at the end of their programs. The supposed advantage of teaching experience in the case of the alternate route teachers did not emerge as a significant factor in our analyses.

It is striking that, despite apparently dramatic structural and philosophical differences between university-based and alternate route programs, so much remains the same about these novice mathematics teachers. What we suspect is that what prospective teachers bring to teaching mathematics from their experiences as students presses powerfully toward sameness--whether they are making a midcareer switch into teaching or whether they are 20-year-old mathematics majors preparing to teach at a university (Ball, 1988; Cohen, 1988). Laced with algorithmic understandings of subject matter and traditional conceptions of pedagogy, this sameness is less a matter of the intellectual caliber or personal traits of the teacher than it is a product of the predictable mathematics education in which they have been steeped.

What it takes to become a good mathematics teacher--one who can teach high school students to understand, care about, and be able to use mathematics--requires thoughtful and powerful interventions. Novice teachers, themselves the products of traditional mathematics classrooms, need to revisit and extend their own mathematical understandings. They need opportunities to examine and challenge their assumptions about the teacher's role, as well as to develop pedagogical content knowledge. And they need opportunities to see and experiment with practices designed to help students learn.

Making an impact on these understandings and assumptions is not so much a matter of the setting in which teacher preparation occurs or the structures that frame its components as it is the nature of what the teacher educators try to do. Teacher educators must turn their attention to the content and pedagogy of teacher education. This includes considering the prior knowledge and beliefs that prospective teachers bring with them, as well as the models those novice teachers have to reflect upon and learn from. Unless they acknowledge the influence of prospective teachers' pedagogical and mathematical biographies, it is unlikely that we will be able to alter the continuity of traditional mathematical teaching and learning.

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